THE EFFECT OF PACKING OF THE RED CELLS ON THE ESR

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The elucidation of the laws governing the sedimentation of the red cells helps toward an understanding of the ESR, a reaction widely used in the practice of medicine and one of considerable diagnostic value.

Among the various factors influencing the rate of sedimentation of the red cells, we may pick out the packing of the red cells. This factor is constantly present in any blood; it plays an essential role in the process of sedimentation of the red cells and appreciably affects the character of the curve of sedimentation of the red cells with time. In the course of sedimentation, as the column of red cells shrinks, their density increases and this interferes with further sedimentation. This factor, which at first is of little consequence, acquires ever increasing importance until the end of the process.

Many processes are characterized by the same feature: with a change in a certain parameter, its further change in the same direction is rendered more difficult. All such processes obey an exponential relationship. We considered it to be possible to consider the changes in the sedimentation of red cells with time as an exponential relationship also. We shall derive a mathematical relationship between the value of the ESR and time.

It can be assumed that

$$v = \frac{c}{\rho}, \qquad (1)$$

where v is the rate of sedimentation of the red cells, ρ the viscosity of the blood, and c is a coefficient.

Now

$$\rho = \frac{y}{h}, \qquad (2)$$

where y is the relative viscosity of the blood and h the height of the column of red cells at given time t.

Substituting the value of ρ from equation (2) in (1), we find

$$v = \frac{c}{y} h$$

or, combining c and y in one coefficient k, specific for each sample of blood, we obtain

$$V = kh. \qquad (3)$$

But the height of the column of red cells

$$h \quad H - S, \tag{4}$$

where H is the initial height of the column of blood (in a capillary tube H = 100 mm), and S is the height of the column of plasma, i.e., the value of the ESR itself.

Therefore,

$$v = k(H - S). \tag{5}$$

Since

$$v = \frac{dS}{dt}, \text{ then } \frac{dS}{dt} = k(H - S). \tag{6}$$

Transposing, we have

$$\frac{dS}{H-S} = kdt. \tag{7}$$

Since

$$d(H-S) = 0 - dS, \text{ then}$$
$$dS = -d(H-S).$$

Substituting in equation (7), we obtain

$$-\frac{d(H-S)}{H-S} = kdt. \tag{8}$$

Integrating,

$$-\ln(H-S) = kt + k_2 \tag{9}$$

Substitute

$$k_2 = -\ln k_3,$$

Then

$$\ln(H-S) + \ln k_3 = -kt. \tag{10}$$

Potentiating equation (10):

$$k_3(H-S) = e^{-kt}; \quad H - S = \frac{1}{k_3} e^{-kt}; \quad S = H - k_4 e^{-kt} \tag{11}$$

Let us deduce the value of k_4. When t = O and S = O, k = H.

Consequently,

$$S = H(1 - e^{-kt}). \tag{12}$$

As the height in equation (12) we must not use the initial height of the column of blood but that of the column of plasma over the fully sedimented red cells in these conditions:

$$S_0 = H - h_0, \tag{13}$$

where h_0 is a constant, unchanged in the particular conditions of sedimentation of the column of red cells.

Consequently,

$$S = S_0 (1 - e^{-kt}). \tag{14}$$

In order to test the applicability of the theoretically derived equation (14) to the ESR – time curve obtained experimentally, we carried out experiments in which the sedimentation in 24 samples of blood was measured by the usual technique every 15 minutes for a period of 2 hours and then at less frequent intervals for 48 hours. At this time the process of sedimentation was practically complete, and a column of red cells of constant height h_0 was obtained; knowing the height of this column, we could calculate the coefficient k for this blood, using an arbitrarily selected value of S at any one of the times at which measurements were taken. Having determined k, the theoretical values of S could be calculated (S_T). We compared these with the experimental values S_{exp}.

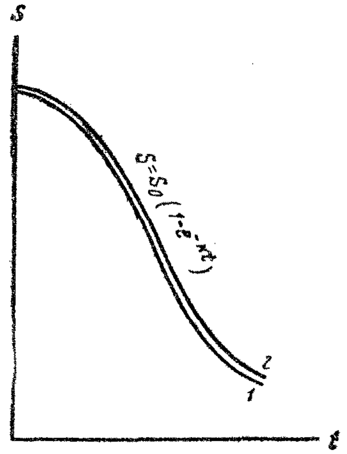

Fig. 1. Experimental (1) and theoretical (2) ESR – time curve when the ESR is only slightly raised.

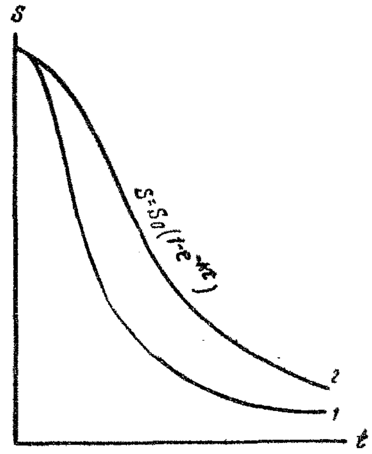

Fig. 2. Experimental (1) and theoretical (2) ESR – time curve when the ESR is raised.

The applicability of equation (14) to the ESR – time curves was tested in the 24 experiments. Examples of certain changes in the sedimentation of red cells with time, as determined experimentally, are compared in the table with the values of S_T calculated theoretically from equation (14). The comparison showed that in the majority of cases when the ESR was normal or slightly raised, the theoretical values S_T agreed reasonably closely with the values S_{exp} obtained experimentally (of the 15 ESR – time curves 14 showed close agreement and one rather more distant agreement). Conversely, in the majority of cases when the ESR was raised, essential discrepancies were observed between the theoretical and experimental values, these being most pronounced in the initial stage of the ESR–time curve. Of 9 cases in which the ESR was raised (from 13 to 56 mm in 1 hr) close agreement was obtained in only 2 cases, and more distant in seven.

We account for the results obtained as follows. The initial portion of the ESR – time curve shows individual peculiarities. It owes its character mainly to the degree of agglomeration of the red cells, and this depends in particular, on the state of the body, and it is increased in pathological processes when the sedimentation rate of the red cells is raised. The final portion is common to any ESR – time curve. It mainly depends on the number of red cells in the blood. When the ESR is nor raised, the degree of agglomeration of the red cells is smaller; it produces no great deviation from the theoretical curve, for which the equation contains no agglomeration factor; the experimental and theoretical ESR – time curves agree (Fig. 1). Conversely, when this agglomeration

TABLE

Applicability of the Equation $S = S_0 (1 - e^{-kt})$ to the ESR – time Curve

Indices	Time (in minutes)	Sedimentation		Difference
		experimental	theoretical	
		in mm		
Cases of close agreement				
1. Limiting sedimentation S_0 = 35 mm; time chosen for finding k, t_k = 720; sedimentation at time t_k, S_k = 28 mm; k = 0.00219	15	0,5	1.2	+0.7
	30	1	2.2	+1.2
	45	3	3.4	+0.4
	60	4	4.3	+0.3
	75	5	5.9	+0.9
	90	7	6.3	–0.7
	105	8	7.2	–0.8
	120	9	8.1	–0.9
	720	28	27.8	–0.2
	1 440	32	33.6	+1.6
	2 880	35	34.9	–0.1
2. S_0 = 56 мм; $t_к$ = 120; $S_к$ = 32 мм; $к$ = 0.007	15	2	5.6	+3.6
	30	6	10.7	+4.3
	45	10	15.8	+5.8
	60	17	19.3	+2.3
	75	2	22.8	+0.8
	90	25	26.2	+1.2
	105	28	29.1	+1.1
	120	32	31.9	–0.1
	135	34	34.2	+0.2
	150	36	35.5	–0.5
	165	37	38.4	+1.4
	180	38	40.1	+2.1
	420	46	52.6	+6.6
	720	50	55.4	+5.4
	1 440	53	56.9	+3.9
	1 880	55	56.9	+1.9
Cases of more distant agreement				
3. S_0 = 60 мм; $t_к$ = 720; $S_к$ = 56 мм; $к$ = 0.00375	15	3	3.3	+0.3
	30	10	6.4	–3.6
	45	19	9.2	–9.8
	60	25	12.0	–13.0
	75	32	14.6	–17.4
	90	37	17.2	–19.8
	105	40	19.5	–20.5
	120	41	21.8	–19.2
	720	56	56.0	0.0
	1 440	58	59.6	+1.6
	2 880	60	59.9	–0.1

is considerable (which is more common when the ESR is raised), a considerable divergence is observed between the experimental and theoretical ESR – time curves, for the latter takes no account of the agglomeration factor (Fig. 2).

The experimental results thus show that packing of the red cells in many cases considerably affects the

ESR — time curve. As a rule this is of no clinical importance. Packing of the red cells, however, conceals and distorts the manifestation of agglomeration of the red cells, which alters the ESR in accordance with the state of the body. It is therefore desirable to develop a method of estimation of the ESR in which packing of the red cells would not take place or would be minimized.

SUMMARY

As the sedimentation of red cells goes on they become increasingly compact, which hinders their further sedimentation. This enables one to apply the exponential relation $S = S_0(1-e^{-kt})$ to the ESR — time curve.

Experimental verification of the formula has demonstrated that with a normal and slightly accelerated ESR there is a good accord between the experimental and the theoretical sedimentation values, while in the event of increased ESR, there is considerable discrepancy between them.

This is caused by the absence of the factor of the red cell agglomeration in the formula, which is of decisive significance in the acceleration of ESR.